Chladni Figures: A Mathematical Exploration of Visual Music

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What phenomena lead to the formation of Chladni figures?

Chladni figures are a way to visualize standing waves on a plate or membrane. By spreading a fine grain such as sand or salt on a two dimensional surface and letting it vibrate, we can observe an agglomeration of the sand along the nodal lines, creating intricate patterns. After first encountering this phenomenon in the context of violin making, I became interested in the underlying physical and mathematical concepts governing these seemingly magical shapes. This paper is my attempt to develop my own mathematical description of Chladni figures with as little external help as possible, starting with only some basic knowledge about differential equations. I then concluded my work by conducting experiments to test my theories.

2-Dimensional Partial Differential Wave Equation

$$\frac{\partial^2 u}{\partial t^2} = \alpha^2 \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right)$$

"Separation of Variables"

Rectangular Plate Solution

$$u(x,y,t) = \sin\left(\frac{n\pi x}{l}\right) \sin\left(\frac{m\pi y}{h}\right) \sin\left(t\pi\alpha\sqrt{\frac{n^2}{l^2} + \frac{m^2}{h^2}}\right)$$

Circular Plate Solution

$$u(x,y,t) = \sin\left(\frac{g\pi x}{\cos((2k-1)\frac{\pi}{2n}) \cdot \frac{r}{\cos(\frac{\pi}{2n})}}\right) \sin\left(\frac{g\pi y}{\sin((2k-1)\frac{\pi}{2n}) \cdot \frac{r}{\cos(\frac{\pi}{2n})}}\right) \cdot$$
$$\sin\left(\frac{g\pi\alpha t}{\sqrt{(\cos((2k-1)\frac{\pi}{2n}) \cdot \frac{r}{\cos(\frac{\pi}{2n})})^2 + (\sin((2k-1)\frac{\pi}{2n}) \cdot \frac{r}{\cos(\frac{\pi}{2n})})^2}}\right)$$

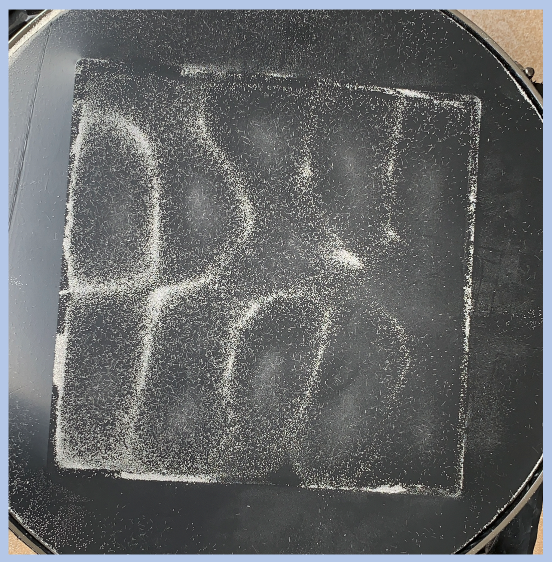
My function for a circular plate is a solution attained through a polygon approximation of a circle. By connecting the center of the polygon with the corner points and treating these lines as vibrating strings, we are able to approximate a circular membrane as the number of corners goes to infinity.

Alternative Solution in Cylindrical Coordinates

Experimental Setup

In order to create my own Chladni figures, I stretched a plastic bag over a drum frame. The membrane was then agitated using a speaker attached to a frequency generator.

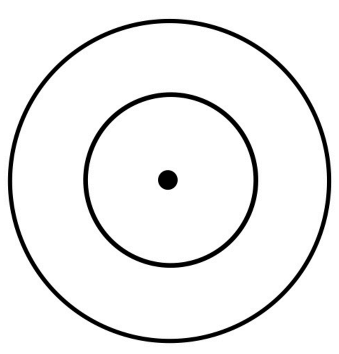
Rectangular Plate Experiments



Circular Plate Experiments

Two of the figures I obtained from my experiments

Comparison of the Theoretical and Experimental Results

Theoretical Expectation	Experimental Result
 mode: g=2	 mode: g=2

Due to the many assumptions and simplifications made in the theoretical model, many figures obtained during the experimental phase were not coherent with theoretical predictions. However, some figures, such as the one on the right, fit quite nicely with theoretical expectations. In conclusion, I definitely succeeded in understanding more about how Chladni figures form, both through mathematical and experimental research.

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